Exercise VII, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

Problems on Mapping Reductions

1 Show that $\leq_m$ is a transitive relation on languages, i.e., if $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$.

2 Show that if A is a Turing-recognizable language and $A \leq_m \overline{A}$, then A is decidable.

3* Show that a language $A \subseteq \{0,1\}^*$ is decidable if and only if $A \leq_m \{0^a1^b \mid a, b \geq 0\}$.

4 A *useless state* in a Turing machine is a state that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

Problems on Time Complexity

5 Arrange the following functions in increasing order according to asymptotic growth.

$$2^n, \quad \binom{n}{5}, \quad (\log\log n)^{10}, \quad n^{300}, \quad \sqrt{n}, \quad (\log n)^n, \quad n/\log n, \quad 2^{2^n}, \quad n^{\sqrt{n}}, \quad n!, \quad \log n$$

6* We call an undirected graph $G = (V, E)$ *Eulerian* if there is a closed walk (i.e. returning to where it starts) on G that uses each edge exactly once. Show that the language

$$\text{EULERIAN-GRAPH} = \{\langle G \rangle : G \text{ is Eulerian}\}$$

is in **P** by giving an algorithm and bounding its running time as a polynomial in $|V|$ and $|E|$.

Hint: *First prove that a connected graph is Eulerian iff every vertex has even degree.*

7 Which of the following operations is the complexity class **P** closed under?

7a Union

7b Intersection

7c Complementation

7d* Kleene closure